

Computer Architecture Midterm

NAME: _____

Read carefully, write legibly, check work, and complete in 1 hour. Good luck!

1 Number representation (20%)

Convert these numbers into the requested base(s).

1. 002537336765473326253 in binary, and then from binary to hexadecimal.
2. -1110 (negative one thousand one hundred ten) in binary using two's complement.
Show place values.

2 Binary arithmetic (20%)

Perform arithmetic in binary. *Show place values and carry bits.*

1.
$$\begin{array}{r} 1011111111 \\ - \underline{11101101} \end{array}$$

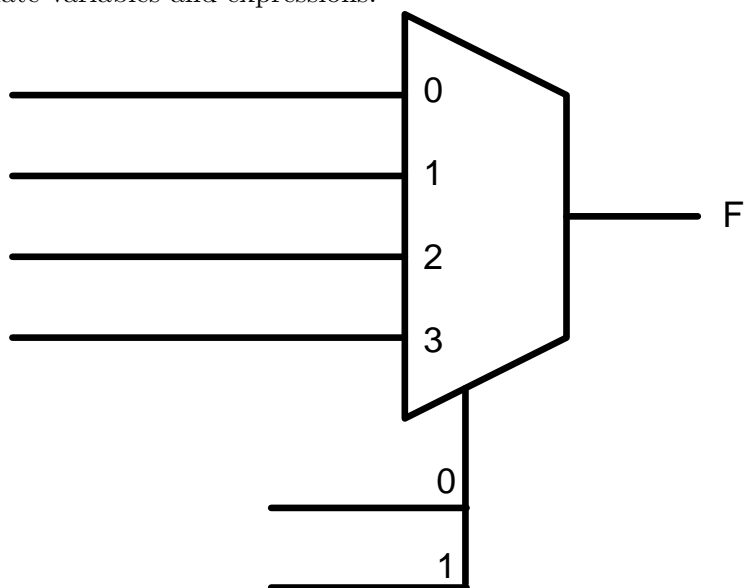
2.
$$\begin{array}{r} 1000010001 \\ + \underline{10011001} \end{array}$$

3 Circuit design (60%)

Given two-bit binary numbers A and B , design a circuit that determines whether $A < B$. For example, since $1 < 3$, when $A = 0b01$ and $B = 0b11$, the circuit should output 1.

1. Draw the truth table for this circuit.
Label inputs as A_1, A_0, B_1, B_0 . Label the output as F .

2. Implement this circuit using a 4-1 MUX. Label the input and select lines with appropriate variables and expressions.



3. Write out the logic expression for this circuit. Simplify and cite laws.

$F =$

4 Laws of Boolean algebra

Law	Form	Dual form
Identity	$a \cdot 1 = a$	$a + 0 = a$
Identity	$a \cdot 0 = 0$	$a + 1 = 1$
Commutative	$a \cdot b = b \cdot a$	$a + b = b + a$
Associative	$(a \cdot b) \cdot c = a \cdot (b \cdot c)$	$(a + b) + c = a + (b + c)$
Distributive	$a \cdot (b + c) = a \cdot b + a \cdot c$	$a + (b \cdot c) = (a + b) \cdot (a + c)$
Idempotence	$a \cdot a = a$	$a + a = a$
Absorption	$a + a \cdot b = a$	$a \cdot (a + b) = a$
Complement	$\overline{0} = 1$	$\overline{1} = 0$
Complement	$a \cdot \overline{a} = 0$	$a + \overline{a} = 1$
Involution	$\overline{\overline{a}} = a$	
DeMorgan's	$\overline{a + b} = \overline{a} \cdot \overline{b}$	$\overline{a \cdot b} = \overline{a} + \overline{b}$
XOR	$a \oplus b = \overline{a} \cdot b + a \cdot \overline{b}$	
XNOR	$\overline{a \oplus b} = \overline{a} \cdot \overline{b} + a \cdot b$	

5 Bonus (5%)

Using DeMorgan's law, show how $\overline{\overline{a} \cdot b + a \cdot \overline{b}}$ simplifies into $\overline{a} \cdot \overline{b} + a \cdot b$.